DTP/95/13

Gravity, Energy Conservation and Parameter Values in Collapse Models

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March, 1995

Abstract

We interpret the probability rule of the CSL collapse theory to mean that the scalar field which causes collapse is the gravitational curvature scalar with two sources, the expectation value of the mass density (smeared over the GRW scale a) and a white noise fluctuating source. We examine two models of the fluctuating source, monopole fluctuations and dipole fluctuations, and show that these correspond to two well known CSL models. We relate the two GRW parameters of CSL to fundamental constants, and we explain the energy increase of particles due to collapse as arising from the loss of vacuum gravitational energy. It is shown how a problem with semi-classical gravity may be cured when it is combined with a CSL collapse model.

1. Introduction

In collapse models the Schrödinger equation is modified so that it describes collapse of the wavefunction as a dynamical process. This modification is introduced for the best possible reason, namely, that the current theory is unable to account properly for experiments. In this respect the inadequacy of standard quantum theory is subtle: the perfect agreement with all present experiments is only obtained through a crude "instantaneous collapse" prescription, which has no explanation within the theory.[1]

In the Continuous Spontaneous Localisation (CSL) theory [2−4] (see section 2), collapse is caused by interaction of the quantum system with a classical scalar

field, $w(\mathbf{x},t)$. The theory probably gives the best description of collapse available at the present time but, inevitably, although it solves some problems it produces others. In particular, it has been criticised for three reasons. The first is that the collapse narrows wavefunctions, thereby producing an increase of energy[5] (see section 3), which raises the question as to whether there is a violation of energy conservation, or whether this energy has some as yet unspecified source.

The second criticism is that the nature of the important physical field w is not specified and, in particular, it is not associated with anything else in physics. Related to this is the third criticism, which is that the two parameters which specify the model are ad hoc. These two parameters, which were originally introduced in the seminal work of Ghirardi, Rimini and Weber,[6] are a distance scale, $a \simeq 10^{-5}$ cm, characterising the distance beyond which the collapse becomes effective, and a time scale, $\lambda^{-1} \simeq 10^{16}$ sec, giving the rate of collapse for, say, a proton. If collapse is a fundamental physical process related to other fundamental processes, it might be expected that the parameters can be written in terms of other physical constants.

In this note we shall address all three of these problems.

There are two equations which characterize CSL. The first equation is a modified Schrödinger equation, which expresses the influence of an arbitrary field $w(\mathbf{x},t)$ on the quantum system. But it is the second equation which stimulates this paper. This equation is a probability rule which gives the probability that nature actually chooses a particular $w(\mathbf{x},t)$. However, this probability rule can be interpreted as expressing the influence of the quantum system on the field $w(\mathbf{x},t)$. It can be shown[3] (section 2) to be completely equivalent to

$$w(\mathbf{x},t) = w_0(\mathbf{x},t) + < A(\mathbf{x},t) > \tag{1.1}$$

where $< A(\mathbf{x},t) >$ is the quantum expectation value of the mass operator smeared over the distance a (see Eq. (2.2)), and $w_0(\mathbf{x},t)$ is a gaussian randomly fluctuating field with zero drift, temporally white noise in character and with a particular spatial correlation function.

In this paper we shall take Eq. (1.1) seriously, using it as a guide to understanding how the collapse formalism fits into the rest of physics. First, it tells us that $w(\mathbf{x},t)$'s average value is $< A >$, a mass density. We therefore are led to write $w(\mathbf{x},t)$ as

$$w(\mathbf{x},t) \equiv \frac{1}{4\pi G}\nabla^2\phi(\mathbf{x},t) \tag{1.2}$$

We note that if we interpret the ϕ defined in (1.2) as the actual gravitational potential (so w is the Newtonian limit of $\frac{1}{2}$ the spacetime curvature scalar), we are doing two unconventional things.

First, we are using semi-classical gravity[7] (because it is the expectation value of A that is the source of ϕ).

Moreover, because of the nature of A, we are led to the strange notion that a point particle has an effect on the gravitational potential as if its mass were smeared over the GRW scale a. We do not believe there is any experimental evidence which conflicts with such a possibility, so we shall entertain it. Indeed, we shall show how (see section 6) a collapse model combined with such a smearing can eliminate a possible source of inconsistency in semi-classical gravity[2] by

ensuring that a nonlocalized state collapses to a localized state before the gravitational field of the nonlocalized state can be measured (essentially, the smearing weakens the gravitational field so that its measurement is prolonged beyond the collapse time).

With this interpretation of w, the presence of w_0 in Eq. (1.1) informs us that the gravitational potential (and associated curvature) also fluctuate. We shall investigate two different naive classical models of the source of these fluctuations, a monopole model (section 4) and a dipole model (section 5).

In the monopole model we assume that the source consists of particles of mass μ which appear at random times and positions: each persists for a short fixed time interval $\mathcal{T}$ in a fixed volume $\mathcal{L}^3$ which we refer to as a "cell." We shall also assume that there is a fixed background of negative mass so that, in each cell, the fluctuations appear as positive and negative masses which average to zero. (This is vaguely like a classical model of virtual quantum fluctuations, with the negative background being a renormalization subtraction.) We shall eventually take μ to be the planck mass ($\mu \equiv (\hbar c/G)^{\frac{1}{2}} \approx 2.2 \times 10^{-5}$ gm.) and $\mathcal{T}$ to be the planck time (so that $\mathcal{T} = \hbar/\mu c^2$ as expected for a quantum fluctuation).

In the dipole model we assume that masses μ and $-\mu$ appear at random but simultaneously in pairs which occupy adjacent cells with random orientation.

Having produced some assumptions which make a well defined break with normal physics, we proceed to discuss the consequences using normal (classical) physics. We calculate the correlation function of w. We find that the correlation function for the monopole model is $\sim \delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$, and the correlation function for the dipole model is $\sim \nabla \cdot \nabla' \delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$. These are respectively identical in form to the correlation functions obtained in the original CSL model[2-4] (hereafter called GRWP), and in the model proposed by Diosi[8] and corrected by Ghirardi, Grassi and Rimini[9] (hereafter called DGGR). Upon equating the correlation functions of the monopole model and GRWP we obtain one equation relating the GRW parameters and the parameters of the monopole model, and likewise for the dipole model and DGGR.

The fluctuating monopoles or dipoles exert a random gravitational force on a particle, causing it to undergo undamped Brownian motion, with the result that, on average, its kinetic energy increases linearly with time.[10] An average linear increase of kinetic energy for each particle is precisely the behavior CSL predicts to occur during collapse. Thus, although we do not have a classical picture of the complete collapse, we have a classical picture for this aspect of it. Upon equating the classical and CSL expressions for the energy increase, we obtain a second equation relating the GRW parameters and the parameters of the monopole model, and likewise for the dipole model. We shall examine the consequences of these two equations for the GRW parameters, for each model.

Thus we have a more—or—less plausible argument that the collapse mechanism is gravitationally related, that the energy for collapse comes from the vacuum, and that the GRW parameters can be related to fundamental constants.

We expect that the reader will, as we do, take the argument given here with a grain of salt. For example, it is very likely that a better justified picture of curvature fluctuations can be found. But, at the least, our arguments indicate that the aforementioned criticisms of CSL are not insurmountable and that

their solutions may point the way toward a more complete theory. At most, it is not inconceivable that the scale of fluctuations necesary for collapse as well as other elements given here may survive in such a theory. Indeed, ideas of earlier authors appear here. Karolyhazy[11] proposed that metric fluctuations play a role in collapse. Penrose[12] has for many years argued that gravity and collapse are linked. Diosi[8] was the first to write an expression for a GRW parameter in terms of fundamental constants in a CSL-gravitational model: he had hoped to do without the mass smearing whose necessity was pointed out by Ghirardi, Grassi and Rimini.[9] The possibility that collapse might cure a crucial problem of semi-classical gravity, which was suggested in the first paper on CSL,[2] is further developed here.

2. CSL

We shall begin with the solution of the general nonrelativistic CSL Schrödinger evolution equation for the statevector in the interaction picture, under the influence of an arbitrary scalar field w($\mathbf{x}$,t):[13]

$$|\psi, T>_w = \mathbf{T}e^{-\int_0^T dt \int\int d\mathbf{x}d\mathbf{x}'[w(\mathbf{x},t)-A(\mathbf{x},t)]G^{-1}(\mathbf{x}-\mathbf{x}')[w(\mathbf{x}',t)-A(\mathbf{x}',t)]}|\psi, 0> \quad (2.1)$$

($\mathbf{T}$ is the time ordering operator). In Eq. (2.1), for fixed t, $A(\mathbf{x}, t)$ is an $\mathbf{x}$-parameter labelled family of commuting (interaction picture) operators toward whose joint eigenstates the collapse tends during the interval $(t,\ t + dt)$. In light of recent discussions of experiments,[14,15] we take A to be the mass density operator (smeared over a):

$$A(\mathbf{x}, t) \equiv e^{iHt}e^{\frac{1}{2}a^2\nabla^2}M(\mathbf{x})e^{-iHt} \quad (2.2)$$

where the mass density operator is

$$M(\mathbf{x}) \equiv \sum_j m_j\xi_j^\dagger(\mathbf{x})\xi_j(\mathbf{x}) \quad (2.2a)$$

($\xi_j(\mathbf{x})$ is the annihilation operator for a particle of mass m_j at the point $\mathbf{x}$) and the smearing is described by

$$e^{\frac{1}{2}a^2\nabla^2}M(\mathbf{x}) = \frac{1}{(2\pi a^2)^{\frac{3}{2}}}\int d\mathbf{z}e^{-\frac{1}{2a^2}(\mathbf{x}-\mathbf{z})^2}M(\mathbf{z}) \quad (2.2b)$$

It is crucial that the mass density operator $M(\mathbf{z})$ be smeared over the scale a, as indicated in Eq. (2.2b). Without such a smearing the energy excitation of particles undergoing collapse would be beyond experimental constraints.

In Eq. (2.1), $G^{-1}(\mathbf{x} - \mathbf{x}')$ is a real positive definite function of $|\mathbf{x} - \mathbf{x}'|$. In later sections we shall be concerned with two particular examples:

$$\begin{aligned} &\text{GRWP:} \qquad G^{-1}(\mathbf{x}-\mathbf{x}') = \gamma\delta(\mathbf{x}-\mathbf{x}') \\ &\text{DGGR:} \qquad G^{-1}(\mathbf{x}-\mathbf{x}') = \gamma'\frac{1}{|\mathbf{x}-\mathbf{x}'|} \end{aligned} \quad (2.3a, b)$$

where γ or γ' is a constant. For later purposes it is useful to define here the inverse of $G^{-1}(\mathbf{x}-\mathbf{x}')$, namely $G(\mathbf{x}-\mathbf{x}')$:

$$\int d\mathbf{z} G(\mathbf{x}-\mathbf{z})G^{-1}(\mathbf{z}-\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}') \tag{2.4}$$

It follows from Eq. (2.4) that the (positive real) fourier transforms of G and G^{-1} are reciprocals. For the two cases,

$$\begin{aligned} &\text{GRWP:} \qquad G(\mathbf{x}-\mathbf{x}') = \frac{1}{\gamma}\delta(\mathbf{x}-\mathbf{x}') \\ &\text{DGGR:} \qquad G(\mathbf{x}-\mathbf{x}') = -\frac{1}{4\pi\gamma'}\nabla^2\delta(\mathbf{x}-\mathbf{x}') \end{aligned} \tag{2.5a,b}$$

The probability rule of CSL, giving the probability that nature chooses a particular $w(\mathbf{x},t)$ for $0 \le t \le T$, is

$$\text{Prob}\{w(\mathbf{x},t)\} = Dw \, {}_w\!< \psi, T|\psi, T >_w \tag{2.6}$$

where Dw is the functional integral element

$$Dw \equiv C \prod_{\text{all } \mathbf{x}: \ t=0}^{t=T} dw(\mathbf{x},t) \tag{2.6a}$$

and C, proportional to $(\det G)^{-\frac{1}{2}}$, makes the integrated probability =1.

Starting with the probability rule (2.6) we may take the following steps, justified because the terms we drop are of negligible order in dt (i.e., they make no contribution to the expectation values of functionals of w):

$$\begin{aligned} &\text{Prob}\{w(\mathbf{x},t)\} = \\ &\quad Dw \prod_{t=0}^{T} \frac{{}_w\!< \psi, t|e^{-2dt\int d\mathbf{x}d\mathbf{x}'[w(\mathbf{x},t)-A(\mathbf{x},t)]G^{-1}(\mathbf{x}-\mathbf{x}')[w(\mathbf{x}',t)-A(\mathbf{x}',t)]}|\psi, t >_w}{{}_w\!< \psi, t|\psi, t >_w} \\ &\qquad = Dw \prod_{t=0}^{T} e^{-2dt\int d\mathbf{x}d\mathbf{x}' w(\mathbf{x},t)G^{-1}(\mathbf{x}-\mathbf{x}')w(\mathbf{x}',t)} \\ &\qquad\qquad \cdot \Big[1 + 4dt\int d\mathbf{x}d\mathbf{x}' G^{-1}(\mathbf{x}-\mathbf{x}')\frac{{}_w\!< \psi, t|A(\mathbf{x}',t)|\psi, t >_w}{{}_w\!< \psi, t|\psi, t >_w}\Big] \\ &\qquad = Dw e^{-2\int_0^T dt\int\int d\mathbf{x}d\mathbf{x}'[w(\mathbf{x},t)-<A(\mathbf{x},t)>]G^{-1}(\mathbf{x}-\mathbf{x}')[w(\mathbf{x}',t)-<A(\mathbf{x}',t)>]} \end{aligned} \tag{2.7a,b,c}$$

$$< A(\mathbf{x},t) > \equiv \frac{{}_w\!< \psi, t|A(\mathbf{x},t)|\psi, t >_w}{{}_w\!< \psi, t|\psi, t >_w} \tag{2.7d}$$

It follows from Eq. (2.7c) that the probability rule is completely equivalent to the expression (1.1) for $w(\mathbf{x},t)$, where $< A(\mathbf{x},t) >$ is given by (2.7d), and $w_0(\mathbf{x},t)$ is a zero drift gaussian process characterized by the correlation function

$$< w_0(\mathbf{x},t)w_0(\mathbf{x}',t) > = \frac{1}{4}G(\mathbf{x}-\mathbf{x}')\delta(t-t') \tag{2.8}$$

The density matrix may be found from Eqs. (2.1), (2.6):

$$\rho(T)=\int Dw\,{}_w\!<\psi,T|\psi,T>_w\frac{|\psi,T>_w\,{}_w\!<\psi,T|}{{}_w\!<\psi,T|\psi,T>_w}=\int Dw|\psi,t>_w\,{}_w\!<\psi,t|$$

$$=\mathbf{T}e^{-\frac{1}{2}\int_0^T dt\int\int d\mathbf{x}d\mathbf{x}'[A(\mathbf{x},t)\otimes 1-1\otimes A(\mathbf{x},t)]G^{-1}(\mathbf{x}-\mathbf{x}')[A(\mathbf{x}',t)\otimes 1-1\otimes A(\mathbf{x}',t)]}\rho(0) \quad (2.9)$$

(the notation is $(A\otimes B)C\equiv ACB$, and the $\mathbf{T}$ operator is time-reverse ordering for operators to the right of $\otimes$).

Before concluding this section, we wish to make one further point. It may have occurred to the reader that there is freedom to transform w, with a concommitant transformation of A and G^{-1} so that the exponent in Eq. (2.1) is left unchanged. In particular, one may think of defining $w(\mathbf{x},t)\equiv\exp[a^2\nabla^2/2]w'(\mathbf{x},t)$, with the result that, in (2.1), $w(\mathbf{x},t)$ is replaced by $w'(\mathbf{x},t)$, $A(\mathbf{x}',t)$ is replaced by $M(\mathbf{x}',t)$, and $G^{-1}(\mathbf{x}-\mathbf{x}')$ is replaced by

$$\begin{aligned}G'^{-1}(\mathbf{x}-\mathbf{x}')&\equiv e^{a^2\nabla^2}G^{-1}(\mathbf{x}-\mathbf{x}')\\&=\frac{1}{(4\pi a^2)^{\frac{3}{2}}}\int d\mathbf{z}e^{-\frac{1}{4a^2}(\mathbf{x}-\mathbf{x}'-\mathbf{z})^2}G^{-1}(\mathbf{z})\\&=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d\mathbf{k}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}e^{-a^2k^2}\tilde{G}^{-1}(\mathbf{k})\end{aligned} \quad (2.10a,b,c)$$

This would result in Eq. (1.1) being replaced by $w'=w_0'+M$, where M is the mass density. It would appear that the smeared mass density A to which we attributed significance in section 1 would be unnecessary. What's wrong with this? It is that this transformation cannot be allowed because w_0' is undefined: G', the correlation function of w_0', is equal to the fourier transform of $\exp[a^2k^2]\tilde{G}(\mathbf{k})$ which does not exist.

However, there are transformations which are allowed. For example, $w\sim\nabla^2\phi$ can be replaced by ϕ, $A(\mathbf{x},t)$ by $\sim\nabla^{-2}A(\mathbf{x},t)=-\int d\mathbf{z}A(\mathbf{z},t)/4\pi|\mathbf{x}-\mathbf{z}|$, and $G^{-1}(\mathbf{x}-\mathbf{x}')$ by $\sim\nabla^2\nabla'^2G^{-1}(\mathbf{x}-\mathbf{x}')$. Since the models obtained under such allowed transformations are equivalent we could, for example, discuss fluctuations of ϕ instead of fluctuations of $w=\nabla^2\phi$, but scalar field seems less fundamental than curvature and the CSL expressions are simplest for $w\sim\nabla^2\phi$.

3. Energy Production

We shall calculate the energy production rate which accompanies collapse. By taking the time derivative of Eq. (2.9), we see that ρ satisfies the differential equation

$$\frac{d\rho(t)}{dt}=-\frac{1}{2}\int\int d\mathbf{x}d\mathbf{x}'G^{-1}(\mathbf{x}-\mathbf{x}')[A(\mathbf{x},t),[A(\mathbf{x}',t),\rho(t)]] \quad (3.1)$$

In the position basis (writing $|x>$ for the position eigenvector of all particles), Eq. (3.1) is

$$\frac{d < x|\rho(t)|x' >}{dt} = -\frac{1}{2}e^{i\mathcal{H}t}\sum_i\sum_j \\ [\Phi(\mathbf{x}_i - \mathbf{x}_j) + \Phi(\mathbf{x}'_i - \mathbf{x}'_j) - 2\Phi(\mathbf{x}_i - \mathbf{x}'_j)]e^{-i\mathcal{H}t} < x|\rho(t)|x' > \tag{3.2}$$

where

$$\Phi(\mathbf{x}_i - \mathbf{x}_j) \equiv \frac{m_i m_j}{(4\pi a^2)^{\frac{3}{2}}}\int d\mathbf{z}G^{-1}(\mathbf{z})e^{-\frac{1}{4a^2}[\mathbf{z}-(\mathbf{x}_i-\mathbf{x}_j)]^2} \tag{3.3}$$

so that for the two cases of interest (using Eqs.(2.3)),

$$\begin{aligned} &\text{GRWP:} \qquad \Phi(\mathbf{x}_i - \mathbf{x}_j) = \left\{\frac{\gamma m_i m_j}{(4\pi a^2)^{\frac{3}{2}}}\right\}e^{-\frac{1}{4a^2}[\mathbf{x}_i-\mathbf{x}_j]^2} \\ &\text{DGGR:} \qquad \Phi(\mathbf{x}_i - \mathbf{x}_j) = \left\{\frac{\gamma' m_i m_j}{a\pi^{\frac{1}{2}}}\right\}\frac{1}{|\mathbf{x}_i - \mathbf{x}_j|}\int_0^{|\mathbf{x}_i-\mathbf{x}_j|} dze^{-\frac{1}{4a^2}z^2} \end{aligned} \tag{3.3a,b}$$

We note that the bracketed expressions in Eqs. (3.3a,b) have the dimension of 1/time.

According to Eq. (3.2), the collapse rate for a "pointer," composed of N particles of mass m with $\mathcal{M} \equiv Nm$ and mass density D, in a state which is a superposition of the pointer in two locations separated by a large distance (much greater than a or the pointer size L) is $\sim \sum_{i,j}\Phi(\mathbf{x}_i - \mathbf{x}_j)$. From (3.3) we obtain

$$\begin{aligned} &\text{GRWP:} \qquad \text{Collapse Rate} \sim \frac{\gamma\mathcal{M}^2}{a^3}\ (L < a), \qquad \sim \gamma\mathcal{M}D\ (L > a) \\ &\text{DGGR:} \qquad \text{Collapse Rate} \sim \frac{\gamma'\mathcal{M}^2}{a}\ (L < a), \qquad \sim \frac{\gamma'\mathcal{M}^2}{L}\ (L > a) \end{aligned} \tag{3.4a,b}$$

These results will be useful in section 6.

To find the average rate of energy increase, we multiply Eq. (3.2) by the Hamiltonian $\mathcal{H}$, and take the trace:

$$\frac{d}{dt}\mathrm{Tr}[H\rho(t)] = \sum_j \frac{3\hbar^2}{4m_j a^2}\lambda_j \tag{3.5}$$

$$\lambda_j \equiv \frac{2m_j^2 a^2}{3(2\pi)^{\frac{3}{2}}}\int d\mathbf{k}k^2 e^{-a^2k^2}\tilde{G}^{-1}(k^2) \tag{3.5a}$$

($\tilde{G}^{-1}(k^2)$ is the fourier transform of G^{-1}). λ_j is the GRW collapse rate for a single particle of mass m_j. For our two cases of special interest (2.3a,b), $\tilde{G}^{-1} = \gamma/(2\pi)^{\frac{3}{2}}$ and $(2/\pi)^{\frac{1}{2}}\gamma/k^2$ respectively, so

$$\begin{aligned} &\text{GRWP:} \qquad \lambda_j = \frac{m_j^2\gamma}{(4\pi)^{\frac{3}{2}}a^3} \\ &\text{DGGR:} \qquad \lambda_j = \frac{m_j^2\gamma'}{3\pi^{\frac{1}{2}}a} \end{aligned} \tag{3.6a,b}$$

It is remarkable that the average rate of energy increase (3.5) is independent of the quantum state of the particles (as well as their interaction potential). This makes possible the classical modelling of this energy increase presented in the next two sections.

4. Monopole Model

For simplicity, imagine space partitioned into cubical cells of edge length $\mathcal{L}$, and time divided into intervals of duration $\mathcal{T}$. Let $u_{\alpha,\beta}(\mathbf{z},t)=1$ if a monopole is in the αth cell during the βth interval, for $\mathbf{z}$ in the cell and t in the interval, and $u_{\alpha,\beta}(\mathbf{z},t)=0$ otherwise. Suppose the monopole mass μ uniformly fills the cell when it appears.

Denote by $\mathcal{P}$ the probability that a monopole appears in any cell during the βth interval. In addition, let there be a constant mass density $-\mu\mathcal{P}/\mathcal{L}^3$ throughout space, so that on average there is zero mass in each cell. The potential at $\mathbf{x}$ when t lies in the βth interval, due to all the fluctuating monopoles and the constant mass density, is therefore

$$\phi(\mathbf{x},t)=-G\mu\sum_\alpha\frac{1}{\mathcal{L}^3}\int d\mathbf{z}\frac{[u_{\alpha,\beta}(\mathbf{z},t)-\mathcal{P}]}{|\mathbf{x}-\mathbf{z}|}\tag{4.1}$$

We note that $<\phi(\mathbf{x},t)>=0$, since $<u_{\alpha,\beta}(\mathbf{z},t)>=\mathcal{P}$.

The correlation function of the potential is

$$\begin{aligned}&<\phi(\mathbf{x},t)\phi(\mathbf{x}',t')>=\\ &\quad\delta_{\beta\beta'}\Theta_\beta(t)\Theta_\beta(t')(G\mu)^2\frac{\mathcal{P}(1-\mathcal{P})}{(\mathcal{L})^6}\sum_\alpha\int\int d\mathbf{z}d\mathbf{z}'\frac{\Theta_\alpha(\mathbf{z})\Theta_\alpha(\mathbf{z}')}{|\mathbf{x}-\mathbf{z}||\mathbf{x}'-\mathbf{z}'|}\\ &\quad\approx\delta(t-t')(G\mu)^2\tilde{\mathcal{P}}\int d\mathbf{z}\frac{1}{|\mathbf{x}-\mathbf{z}||\mathbf{x}'-\mathbf{z}|}\end{aligned}\tag{4.2a, b}$$

with dependence only upon the combination of model parameters $\tilde{\mathcal{P}}\equiv\mathcal{P}\mathcal{T}/\mathcal{L}^3$. We have also introduced the characteristic function $\Theta_\alpha(\mathbf{z})$ of the αth cell, which equals 1 if $\mathbf{z}$ lies in the cell, and 0 otherwise. Similarly, $\Theta_\beta(t)$ is the characteristic function for the βth time interval. In Eq. (4.2b) we have made some approximations: $\mathcal{P}<<1$, $\delta_{\beta\beta'}\Theta_\beta(t)\Theta_\beta(t')\approx\mathcal{T}\delta(t-t')$ and $\Theta_\alpha(\mathbf{z})\Theta_\alpha(\mathbf{z}')\approx(\mathcal{L})^3\delta(\mathbf{z}-\mathbf{z}')$.

Eq. (4.2b) is all we shall need to calculate all quantities of interest (actually, the integral on the right hand side of (4.2b) must be cut off at large $|\mathbf{z}|$ to exist, but the quantities we shall calculate need no cutoff). The correlation function of $w_0(\mathbf{x},t)=(4\pi G)^{-1}\nabla^2\phi(\mathbf{x},t)$ is easily obtained:

$$<w_0(\mathbf{x},t)w_0(\mathbf{x}',t')>=\mu^2\tilde{\mathcal{P}}\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')\tag{4.3}$$

We see from Eqs. (2.5a) and (2.8) that these monopole fluctuations give us the GRWP correlation function, with $\gamma=1/4\mu^2\tilde{\mathcal{P}}$. We thus obtain from Eq. (3.6a) an expression for the collapse rate for a particle of mass m:

$$\lambda_m=\frac{m^2}{32\pi^{\frac{3}{2}}\mu^2a^3\tilde{\mathcal{P}}}\tag{4.4}$$

We now turn to calculate the rate of energy increase of a particle of mass m due to the force it feels from the fluctuating potential. As explained in section 1, the particle causes its own gravitational potential as if its mass were smeared over the scale a so, for consistency, when we calculate the force on a particle at the origin, we treat it as a rigid mass distribution of density $m(2\pi a^2)^{-\frac{3}{2}}\exp -x^2/2a^2$. The correlation function of the force follows from Eq. (4.2b):

$$< F^i(t)F^j(t') >= \delta(t-t')\frac{(G\mu m)^2\tilde{\mathcal{P}}}{(2\pi a^2)^3}\int d\mathbf{x}d\mathbf{x}'d\mathbf{z}e^{-\frac{1}{2a^2}(x^2+x'^2)}\partial_i\partial'_j\frac{1}{|\mathbf{x}-\mathbf{z}||\mathbf{x}'-\mathbf{z}|} \tag{4.5a}$$

Using the symmetry of the integrand, $\partial_i\partial'_j$ may be replaced by $-(1/3)\delta_{ij}\nabla_z^2$ which, acting on $|\mathbf{x}-\mathbf{z}|$, gives a delta function. The remaining integral is straightforward, and the result is

$$< F^i(t)F^j(t') >= \delta(t-t')\delta_{ij}\frac{4\pi^{\frac{1}{2}}(G\mu m)^2\tilde{\mathcal{P}}}{3a} \tag{4.5b}$$

The correlation function in Eq. (4.5b) is that of white noise. Therefore we can write $F_i(t) = KdB_i(t)/dt$, where K^2 is the constant factor in (4.5b) and $B_i(t)$ is Brownian motion,

$< B_i(t)B_j(t) >= \delta_{ij}t$. By Newton's second law, the momentum is $KB_i(t)$, so the energy is $E = K^2\mathbf{B}(t)\cdot\mathbf{B}(t)/2m$. We thus obtain

$$\frac{d< E >}{dt} = \frac{2\pi^{\frac{1}{2}}m(G\mu)^2\tilde{\mathcal{P}}}{a} \tag{4.6}$$

The effect of the fluctuating field on the particle, according to both our classical calculation (4.6) and the CSL calculation (3.5), is a linear rate of increase of energy. Equating the two provides a second relationship involving λ_m and a:

$$\frac{3\hbar^2}{4ma^2}\lambda_m = \frac{2\pi^{\frac{1}{2}}m(G\mu)^2\tilde{\mathcal{P}}}{a} \tag{4.7}$$

Eqs. (4.4) and (4.7) may be solved for λ_m and a:

$$\begin{aligned} a &= \left(\frac{3}{\pi^2}\right)^{\frac{1}{4}}\frac{1}{4}\left(\frac{c\hbar}{G\mu^2}\right)^{\frac{1}{2}}\left(\frac{1}{\tilde{\mathcal{P}}c}\right)^{\frac{1}{2}} \\ \lambda_m &= \frac{1}{2(3\pi)^{\frac{1}{2}}}\frac{Gm^2}{a\hbar} \end{aligned} \tag{4.8a, b}$$

The fact that a is independent of m (as required by the CSL models proposed so far), and that λ_m is proportional to m^2 (as required by (3.6a)) may be regarded as modest successes of the model. It is also interesting that $\lambda_m a$ is independent of the parameters of the model.

If we use the GRW value for a (10^{-5} cm) in (4.8b), then we find $\lambda_m \simeq 10^{-24}$ sec^{-1} for a nucleon. While this is eight orders of magnitude smaller than the value given by GRW, it is not completely unreasonable. Indeed, the expression (4.8b) for λ for a proton and the collapse rate for objects of size smaller than a is the same as in DGGR (up to a numerical factor).

The collapse time for a cube .01 cm on a side in a superposition of states with separation larger than .01 cm is longer than in GRWP or DGGR, but still a respectable 10^{-5} sec.

We have no good argument for choosing $\tilde{\mathcal{P}}$ and μ, and so determining a. Nonetheless, it is interesting to see what is implied if we use the planck mass for μ (so we may set $c\hbar/G\mu^2 = 1$ in (4.8a)), the planck time for $\mathcal{T}$, and the GRW value for a. Then, by (4.8a), $\mathcal{P}/\mathcal{L}^3 \approx (4a)^{-2}(\hbar/\mu c)^{-1} \approx 4 \times 10^{41}$ cm^{-3}. It so happens that $1/(\hbar/Mc)^3 \approx 10^{41}$ cm^{-3}, where M is the nucleon mass. Indeed, if we use $\mathcal{P}/\mathcal{L}^3 = 1/(\hbar/Mc)^3$ in Eqs. (4.8) we obtain

$$
\begin{aligned}
a = (\frac{3}{\pi^2})^{\frac{1}{4}} \frac{\hbar}{4Mc}\sqrt{\frac{\mu}{M}} \approx 1.4 \times 10^{-5} \text{ cm} \\
\lambda_m \approx 2 \times 10^{-24} \text{ sec}^{-1} \text{ for the nucleon}
\end{aligned}
\qquad (4.9a, b)
$$

Thus we obtain the GRW value for a provided the frequency of appearance of the monopole "planckons" is such that on average there is always one present per proton volume (taking the compton radius of the proton to characterize its size). This is a very suggestive number even though we have no obvious theory for it. It suggests that the existence of a particle of mass m may cause planckon fluctuations in a region of space around it with probability/vol equal to $1/(\hbar/mc)^3$. Then, in ordinary matter, the probability/vol is dominated by the planckons due to the presence of the nucleons, since the probability/vol due to the presence of electrons is 10^{-10} times smaller. (This smaller planckon production rate would, of course, be obtained in a purely electron plasma). This would make a dependent upon the milieu in which particles find themselves, and would represent a variant of standard CSL, where it has been assumed up to now that a is universal.

5. Dipole Model

We now repeat the calculations of the previous section when a dipole $\mathbf{p}$ appears in the center of a cell, with random orientation. It is convenient to imagine the unit sphere centered on the cell partitioned into small solid angle sections of size $d\Omega$ labelled by γ, with $\mathbf{p}$ only allowed to take on values $\mathbf{p}_\gamma$ (pointing to the center of the γth section). $u_{\alpha,\beta,\gamma}(\mathbf{z}, t) = 1$ is defined as before, with the additional implication that $\mathbf{p} = \mathbf{p}_\gamma$. The potential at $\mathbf{x}$ when t lies in the βth interval is

$$
\phi(\mathbf{x}, t) = -G \sum_{\alpha,\gamma} \frac{1}{\mathcal{L}^3} \int d\mathbf{z} u_{\alpha,\beta,\gamma}(\mathbf{z}, t)\mathbf{p}_\gamma \cdot \nabla_z \frac{1}{|\mathbf{x} - \mathbf{z}|} \qquad (5.1)
$$

We note that $< \phi(\mathbf{x}, t) >= 0$ since all polarization directions are equally probable. Using the same approximations that were made to obtain Eq. (4.2b), except that $\mathcal{P}$ need not be small, the correlation function of the potential is

$$< \phi(\mathbf{x},t)\phi(\mathbf{x}',t') >= $$
$$\delta(t-t')G^2\tilde{\mathcal{P}}\int d\mathbf{z}\int \frac{d\Omega}{4\pi}\mathbf{p}\cdot\nabla_z\frac{1}{|\mathbf{x}-\mathbf{z}|}\mathbf{p}\cdot\nabla_z\frac{1}{|\mathbf{x}'-\mathbf{z}|} \qquad (5.2a,b)$$
$$= \delta(t-t')\frac{4\pi}{3}G^2\tilde{\mathcal{P}}p^2\frac{1}{|\mathbf{x}-\mathbf{x}'|}$$

The correlation function of $w_0(\mathbf{x},t) = (4\pi G)^{-1}\nabla^2\phi(\mathbf{x},t)$ may now be calculated using Eq. (5.2b):

$$< w_0(\mathbf{x},t)w_0(\mathbf{x}',t') >= 3^{-1}\tilde{\mathcal{P}}p^2\delta(t-t')(-\nabla^2)\delta(\mathbf{x}-\mathbf{x}') \qquad (5.3)$$

We see from Eqs. (2.5b) and (2.8) that that these dipole fluctuations give us the DGGR correlation function with

$$\gamma' = \frac{3}{16\pi\tilde{\mathcal{P}}p^2} \qquad (5.3a)$$

We thus obtain from Eq. (3.5b) an expression for the collapse rate for a particle of mass m:

$$\lambda_m = \frac{m^2}{16\pi^{\frac{3}{2}}a\tilde{\mathcal{P}}p^2} \qquad (5.4)$$

Proceeding exactly as in section 4, we now turn to calculate the rate of energy increase of a particle of (smeared) mass m due to the force it feels from the fluctuating potential. The correlation function of the force follows from Eq. (5.2b):

$$< F^i(t)F^j(t') > = \delta(t-t')\frac{4\pi G^2m^2\tilde{\mathcal{P}}p^2}{3(2\pi a^2)^3}\int d\mathbf{x}d\mathbf{x}'e^{-\frac{1}{2a^2}(x^2+x'^2)}\partial_i\partial_j'\frac{1}{|\mathbf{x}-\mathbf{x}'|}$$
$$= \delta(t-t')\delta_{ij}\frac{2\pi^{\frac{1}{2}}G^2m^2\tilde{\mathcal{P}}p^2}{9a^3} \qquad (5.5a,b)$$

The correlation function in Eq. (5.5b) is that of white noise, and we write $F_i(t) = KdB_i(t)/dt$, where K^2 is the constant factor in (5.5b) and $B_i(t)$ is Brownian motion.

The momentum is thus $KB_i(t)$ and the energy is $E = K^2\mathbf{B}(t)\cdot\mathbf{B}(t)/2m$, yielding

$$\frac{d<E>}{dt} = \frac{\pi^{\frac{1}{2}}G^2m\tilde{\mathcal{P}}p^2}{3a^3} \qquad (5.6)$$

Equating (5.6) to the CSL rate of energy increase given by (3.4)
provides our second relationship involving λ_m and a:

$$\frac{3\hbar^2}{4ma^2}\lambda_m = \frac{\pi^{\frac{1}{2}}G^2m\tilde{\mathcal{P}}p^2}{3a^3\hbar} \qquad (5.7)$$

Eqs. (5.4) and (5.7) may be solved for $\tilde{\mathcal{P}}p^2$ and λ_m:

$$\begin{aligned}\tilde{\mathcal{P}}p^2 &= \frac{3}{8\pi}\Big(\frac{\hbar}{G}\Big)\\ \lambda_m &= \frac{1}{6\pi^{\frac{1}{2}}}\frac{Gm^2}{a\hbar}\end{aligned} \tag{5.8a, b}$$

The dipole model gives essentially the same expression for $\lambda_m a$ as does the monopole model (apart from a factor $3^{\frac{1}{2}}$, Eq. (5.8b) is the same as (4.8b)), and hence essentially the same numerical value (4.9b) for λ_m when a is taken as 10^{-5} cm. However, Eq. (5.8a) is a requirement upon $\tilde{\mathcal{P}}p^2$ which is independent of a and λ_m, so this equation gives no further information about the collapse parameters. On the other hand, if we take the "natural" values $\mathcal{T}$ = planck time and $p = \hbar/c$ (the planck mass times planck length), we find the intriguing result

$$\frac{\mathcal{P}}{\mathcal{L}^3} = \frac{3}{8\pi}\frac{1}{(\hbar/\mu c)^3} \tag{5.9}$$

which means that, on average, there is always one dipole in a volume of the order of the planck volume.

6. Consistency of Semi-Classical Gravity?

The results we have obtained are based upon presumption of a connection between collapse and semi-classical gravity which was suggested by Eq. (1.1). In sections 4 and 5 gravity proved fruitful for collapse, and in this section collapse will return the favor.

Perhaps the most controversial aspect of semi-classical gravity, as an approximation to the "true" quantum gravity, arises because the expectation value of the stress tensor is the gravitational source. Consider a sphere of matter of radius R, and let the state $|Z>$ describe the sphere with center on the z-axis at $z = Z$, and suppose the state of the sphere is $(1/\sqrt{2})[|Z> + |-Z>]$. A probe mass moving along the x-axis will, according to the standard nonrelativistic quantum theory of gravity, become entangled with the state of the sphere, resulting in the statevector $(1/\sqrt{2})[|Z>|\mathrm{up}> + |-Z>|\mathrm{down}>]$, where $|\mathrm{up}>$ ($|\mathrm{down}>$) means that the probe mass is deflected in the positive (negative) z-direction. According to semi-classical gravity the probe mass should be undeflected. This was actually tested,[16] with the (not unexpected) result that the mass is deflected.

A theoretical objection to semi-classical gravity is that it allows superluminal communication.[17] To see this, consider the entangled state $(1/\sqrt{2})[|Z>|1> + |-Z>|2>]$, where the states $|1>$ and $|2>$ denote orthogonal states of a system which is a large distance from the sphere, but close to a "sender." If the sender chooses not to measure the system, the "receiver," who is close to the sphere and uses the probe mass as described above, finds it undeflected. If, on the other hand, the sender chooses to measure the system, thereby finding it to be in state $|1>$ or $|2>$, the sphere will immediately be in the state $|Z>$ or $|-Z>$ respectively. Then the receiver will be able to see this because the probe mass will now be deflected up or down.

As we have earlier suggested,[2] these problems would disappear if the superposition $(1/\sqrt{2})[|Z>+|-Z>]$ spontaneously collapses to $|Z>$ or $|-Z>$ before the probe mass can complete the measurement. We shall now investigate the conditions under which this occurs.

First, consider the probe particle. Its uncertainty in position Δz should be of the order of, or less than Z and its mass should be less than the sphere's mass $\mathcal{M}$ in order to obtain an unambiguous deflection indicative of the state of the sphere. Thus the probe's velocity uncertainty satisfies

$$\Delta v_z > \frac{\hbar}{\mathcal{M}Z} \tag{6.1}$$

Now, we are adopting the smearing hypothesis, i.e., the gravitational force exerted by each particle is as if the particle's mass is smeared out over a sphere whose radius we shall call a': no relation between a' and the GRW parameter a is as yet assumed. For maximum deflection of the probe we take Z equal to the sum of the radii of the *effective* spherical mass distributions of the probe and sphere, so

$$Z > \max(R, a') \tag{6.2}$$

If the probe moves with speed w, the time for the measurement to be performed is $\approx Z/w$. From Newton's second law we can find the z-speed of the probe, v_z, if it is deflected by a sphere at Z. The condition for a good measurement, capable of detecting whether the deflection source is one sphere or the superposition, is $v_z > \Delta v_z$:

$$\frac{G\mathcal{M}}{Z^2}\frac{Z}{w} > \frac{\hbar}{\mathcal{M}Z} \text{ or } \left(\frac{\mathcal{M}}{\mu}\right)^2 > \frac{w}{c} \tag{6.3}$$

We emphasize that, although Z does not directly appear in the condition (6.3), it must be restricted as in (6.2) in order that the total mass $\mathcal{M}$ (and not a fraction thereof) be the correct mass to appear in (6.3).

Now, first consider the case $R < a$. Using Eqs. (3.4) and $\gamma \sim Ga^2/\hbar$, $\gamma' \sim G/\hbar$, we find that the collapse rate for both models is the same, $\sim G\mathcal{M}^2/a\hbar$. In order for the outcome of the experiment to

be that the probe is undeflected, the collapse time must be longer than the time it takes to complete the experiment:

$$\frac{a\hbar}{G\mathcal{M}^2} > \frac{Z}{w} \tag{6.4}$$

Combining the inequalities (6.3) and (6.4), we obtain a necessary condition for the successful detection of the sphere in a superposed state:

$$a > Z > a' \tag{6.5}$$

(the second inequality in (6.5) comes from (6.2)). If e.g., $\mathcal{M}$ represents some elementary particle and a' could be e.g., its compton radius, then Eq. (6.5) could easily be satisfied. But if the smearing length a' is chosen equal to a, as is mandated by our collapse models, then the inequality (6.5) cannot be satisfied. Thus, in this case, it is impossible to detect the sphere in a superposition of states

by means of the semi-classical gravitational force exerted by that superposed state.

Lastly, consider the case $R > a$. Using Eq. (3.4) we find the collapse time for the two models. The condition that the collapse time be longer than the time it takes to complete the experiment is

$$\begin{aligned} &\text{GRWP:} \qquad \frac{\hbar}{G\mathcal{M}Da^2} > \frac{Z}{w} \\ &\text{DGGR:} \qquad \frac{\hbar R}{G\mathcal{M}^2} > \frac{Z}{w} \end{aligned} \tag{6.6a, b}$$

Combining the inequalities (6.3) and (6.6), we obtain a necessary condition for the successful detection of the sphere in a superposed state:

$$\begin{aligned} &\text{GRWP:} \qquad \frac{\mathcal{M}}{Da^2} > Z > \max(R, a') \\ &\text{DGGR:} \qquad R > Z > \max(R, a') \end{aligned} \tag{6.7a, b}$$

(the second inequality in Eqs. (6.7) comes from (6.2)). Eq. (6.7a) can be satisfied for a sufficiently massive object, regardless of the choice of a', since $\mathcal{M} \sim R^3$. However, Eq. (6.7b) cannot be satisfied.

Thus we conclude, as far as our tentative exploration of the issue is concerned, that a CSL collapse model based upon dipole fluctuations may very well allow semi-classical gravity to have its cake and eat it too: the metric can be responsive to the expectation value of the stress tensor, yet a nonlocal superposition cannot be detected. Our investigations suggest that it may be worthwhile to look at collapse, aspects of semi-classical gravity, and mass-smearing as possible features of quantum gravity.

Acknowledgments

P. P. would like to thank Henry Stapp for stimulating conversations, and the Hughes foundation for a supporting grant through Hamilton College. E. J. S. would like to thank Chris Dove for discussions.

References and Remarks

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